Unique Paper Code : **62351101**

Name of the Paper : **Calculus**

Name of the Course : **CBCS-LOCF B.A.(Program) I Year**

Semester : **I**

Duration : **3 Hours**

Maximum Marks : **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. (a) Give the ε-δ definition of a limit of a function. Using the same definition , prove that

$$\lim_{x\to c} x^2 = c^2$$

$$f(x) = \begin{cases} \frac{e^{\frac{1}{x}} - e^{\frac{-1}{x}}}{e^{\frac{1}{x}} + e^{\frac{-1}{x}}} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$$

is discontinuous at $x = 0$. Also mention the type of discontinuity. Is differentiable function continuous? Justify your answer.

2. State Leibnitz's theorem for finding the n^{th} differential coefficient of the product of two functions.

If $y = \tan^{-1} x$, prove that $(1 + x^2)y_{n+2} + 2(n + 1)\, x\, y_{n+1} + n(n + 1)y_n = 0$

If $u = \cos^{-1}\left[\frac{x+y}{\sqrt{x}+\sqrt{y}}\right]$, then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$$

3. Prove that the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ touches the straight line $\frac{x}{a} + \frac{y}{b} = 2$ at the point (a, b), whatever be the value of *n.* Show that the normal at any point of the curve

$$x = a\cos t + at\ \sin t\,, y = a\sin t - at\cos t$$

is at the constant distance from the origin

If ρ_1 and ρ_2 are the radii of curvature at the extremities of the focal chord of a parabola

$y^2 = 4ax$, prove that $(\rho_1)^{-\frac{2}{3}} + (\rho_2)^{-\frac{2}{3}} = (2a)^{-\frac{2}{3}}$

4. Prove that the curve $ay^2 = (x-a)^2(x-b)$ has at $x = a$, a conjugate point if $a < b$, a node if $a > b$ and a cusp if a = b.

Find asymptotes of the following curve

$$x^3 - x^2y - xy^2 + y^3 + 2x^2 - 4y^2 + 2xy + x + y + 1 = 0$$

Trace the curve $y^2(a+x) = x^2(3a-x)$

5. State Rolle's Theorem and give its geometrical interpretation. Show that there is no real number k for which the equation $x^3 - 3x + k = 0$ has two distinct roots in [0, 1]

Prove that $\frac{\tan x}{x} > \frac{x}{\sin x}$ for $0 < x < \frac{\pi}{2}$

Use Taylor's theorem to prove that $1 + x + \frac{x^2}{2} < e^x < 1 + x + \frac{x^2}{2}e^x$ for all $x > 0$

6. Find the Maclaurin's Series expansion of the function $f(x) = (1+x)^m$, m is a positive integer.

Evaluate $\lim_{x\to 0}\left(\frac{1}{e^x-1} - \frac{1}{x}\right)$

Find the maximum value of $\left(\frac{1}{x}\right)^x$

Find the value of x at which $\sin x\,(1+\cos x)$ is maximum.