Unique Paper Code	**:**	**62351101**
Name of the Paper	**:**	**Calculus**
Name of the Course	**:**	**CBCS-LOCF B.A.(Prog.) I Year**
Semester	**:**	**I**
Duration	**:**	**3 Hours**
Maximum Marks	**:**	**75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Give the ε-δ definition of the limit of a function. Prove that the limit is unique. Using the ε-δ definition, prove that

$$\lim_{x\to 0} x^2 \sin\frac{1}{x} = 0.$$

Examine the continuity of the function

$$f(x) = \frac{1}{1-e^{\frac{1}{x}}} \text{ at } x = 0.$$

Is continuous function differentiable? Justify your answer.

2. If $y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$, find y_n.

If $y = e^{\tan^{-1}x}$, prove that

$$(1 + x^2)y_{n+2} + (2(n + 1)\, x - 1)y_{n+1} + n(n + 1)y_n = 0.$$

If $u = \log\frac{x^4+y^4}{x+y}$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\ .$$

3. Find the condition that the straight line $p = x\cos\alpha + y\ \sin\alpha$ touches the curve

$$\left(\frac{x}{a}\right)^{\frac{n}{n-1}} + \left(\frac{y}{b}\right)^{\frac{n}{n-1}} = 1$$

Show that the tangent and normal at any point of the curve

$$x = ae^{\theta}(\sin\theta - \cos\theta)\,, y = ae^{\theta}(\sin\theta + \cos\theta)$$

are equidistant from the origin. Prove that portion of the tangent to the curve $x^m y^n = a^{m+n}$ intercepted between the axes is divided in the ratio m: n at the point of contact.

4. Determine the position and nature of double points on the curve

$$x^4 - 4y^3 - 12\,y^2 - 8x^2 + 16 = 0.$$

Find asymptotes of the following curve

$$y^3 - 2xy^2 - x^2y + 2x^3 + 2x^2 - 3xy + x - 2y + 1 = 0\,.$$

Trace the curve $x^2(x^2 + y^2) = a^2(x^2 - y^2)$.

5. State Lagrange's mean value theorem and give its geometrical interpretation. Verify the mean value theorem for the function

$$f(x) = \ x^3 - 6x^2 - 2$$

in the interval $[0\,,2]$.

Show that

$$\frac{2}{\pi} < \frac{\sin x}{x} < 1 \ \text{ for } \ 0 < x < \frac{\pi}{2}$$

Use Taylor's theorem to prove that $1 + x + \frac{x^2}{2} + \frac{x^3}{3!} < e^x < 1 + x + \frac{x^2}{2} + \frac{x^3}{3!}e^x$ for all $x > 0$

6. Find the Maclaurin's Series expansion of the function $f(x) = \sin x,\ x \in R$.

For what value of a does $\frac{\sin 2x + a \sin x}{x^3}$ tend to a finite limit as $x \to 0$.

Show that the function f defined by

$$f(x) = x^5 - 5x^4 + 5x^3 - 1,\ \forall\, x \in R$$

has a maximum value at $x = 1$, a minimum value at $x = 3$ and neither at $x = 0$.