(This question paper contains printed pages)

Roll Number:
Serial Number of question paper:

Unique Paper Code:	**12277502_NC**
Name of the Paper:	**Applied Econometrics**
Name of the Course:	**B.A. (Honours) Economics CBCS**
Semester	**Semester V**
Duration:	**3 hours**
Maximum Marks:	**75**

Instructions for Candidates

- **Write your Roll No. on the top immediately on receipt of this question paper.**
- **Answers may be written in English or Hindi but the same medium should be used throughout the paper.**
- **The question paper consists of six questions. Answer any four questions.**
- **All questions carry equal marks.**
- **Use of simple non-programmable calculators is allowed.**
- **Statistical tables are attached for your reference.**

परीक्षार्थियों हेतु अनुदेश

- इस प्रश्न-पत्र के प्राप्त होते ही तुरन्त सबसे ऊपर अपना रोल नम्बर लिखिए।
- उत्तर अंग्रेजी या हिन्दी में दिए जा सकते हैं परन्तु पूरे पेपर में एक ही माध्यम का उपयोग किया जाना चाहिए।
- इस प्रश्न-पत्र में छः प्रश्न हैं। किन्हीं चार प्रश्नों के उत्तर दीजिए।
- सभी प्रश्नों के बराबर अंक हैं।
- साधारण अप्रोग्रामनीय कैलकुलेटर का प्रयोग मान्य है।
- आपके सन्दर्भ हेतु सांख्यिकीय सारिणियाँ संलग्न हैं।

Q1. (a) To study whether the savings-income relationship has changed over time (monthly), the following regression was estimated –

$$Y_t = \beta_0 + \beta_1 D_t + \beta_2 X_t + \beta_3 D_t X_t + u_t$$

where Y_t = Savings (in Rs), X_t = Income (in Rs) and u_t = stochastic disturbance. The variable D_t is defined as D_t = 1 for the monthly observations in the period 2002-2010 and D_t = 0, for the monthly observations in the period 2011-2019. The regression equation was estimated as –

$\widehat{Y_t}$ = -1.82 + 1.56D_t + 0.18X_t – 0.40$D_t X_t$
SE 0.3322 0.4724 0.0264 0.0311

(i) Interpret the regression coefficients.
(ii) Has the saving income relationship changed between the two
periods? Use 5% level of significance. State the null and alternative hypotheses clearly.
(iii) Explain how recursive least squares and Chow's prediction failure test helps to determine the point of a structural break.

Q1. (b) Consider the following time series regression model:

$$C_t = \beta_1 + \beta_2 X_t^* + u_t$$

where C is private consumption expenditure, X* is permanent income and t is a time-period. X_t^* is not directly observable. Suppose we adopt the adaptive expectations hypothesis for X_t^*.

(i) Write the equation showing how the expectations are formed.
(ii) Using the equation formed in (i), derive the estimated regression model.
(iii) Now from quarterly data for the United States for the period 2000-2019 following regression results were obtained using the estimated model:

$$\widehat{C_t} = 1001.4 + 0.3031 X_t + 0.4009 C_{t-1}$$

se	(2503.43)	(0.0817)	(0.1131)
t-stat	(0.4000)	(3.7099)	(3.5446)

R^2 = 0.99, d = 1.4012, F = 1198.46

What are the short-run marginal propensity and long-run marginal propensity to consume?
(iv) Do the estimates conform with the economic theory? Explain?
(v) How would you test for autocorrelation in the above model? Conduct the appropriate test using $\alpha = 5\%$.

(9.5+9.25)

Q2. (a) In a standard regression model with y = $X\beta + u$ and var(y) = $\sigma^2 I$, and OLS estimator b = $(X'X)^{-1}X'y$

(i) Prove that b is an unbiased estimator of β.
(ii) Derive the expected value and variance of $\hat{\beta}$= b+Ay where A is any (k X n) non-stochastic matrix with AX = 0.
(iii) How does Var ($\hat{\beta}_k$) compare with Var (b_k)? What are the implications of your result?

Q2. (b) (i) Explain, using an example, the reasons for using the method of instrumental variables (IVs).
(ii) What are the necessary conditions an instrumental variable should satisfy?
(iii) When do we say that an IV is poor? Are there any costs of using it?

(9.5+9.25)

Qs. 3 (a) Consider the Linear Probability Model (LPM),

$$\widehat{Y_i} = 1.123 + -0.004age_i - 0.02edu_i - 0.005price_i$$

where $Y_i = 1\ for\ smoker$
$Y_i = 0\ for\ non-smoker$

(i) Explain the linear probability model (LPM) approach to the estimation of the model using OLS. Interpret the regression coefficients.
(ii) Explain the inherent limitations of the model.
(iii) Explain how the logit model can be a solution to the above problems.

Qs. 3 (b) Consider a cross-sectional model of petroleum consumption by state:

$$pcons_i = B_o + B_1 Reg_i + B_2 price_i + \varepsilon_i;\ \text{N=50}$$

where pcons = petroleum consumption in the i^{th} state
reg = motor vehicle registrations in the i^{th} state

price = price of petrol in the i^{th} state

(i) To investigate the possibility of heteroscedasticity caused by variation in the size of the states, explain the steps in Breusch-Pagan methodology.
(ii) If the estimated R^2= 0.197 for the auxiliary regression, what do you conclude? State the null and alternate hypotheses clearly. Use a 5% level of significance.
(iii) If the test shows evidence of heteroscedasticity, then what should be done about it?

(9.5+9.25)

Qs. 4 (a) Consider the following infinite lag model:

$$Y_t = \alpha + \beta_0 X_t + \beta_0 \lambda X_{t-1} + \beta_0 \lambda^2 X_{t-2} + \beta_0 \lambda^3 X_{t-3} + \ldots\ldots\ldots + u_t$$

Where $0<\lambda < 1$

(i) Show how the Koyck transformation can be used to estimate the above model.
(ii) Derive the mean lag and median lag in the model derived in (i).
(iii) Prove that the stochastic disturbance term of the model derived in (i) is serially correlated and also correlated with one year lag of the dependent variable.
(iv) Will the estimates obtained in the model derived in (i) be unbiased and consistent? Why and why not?

Qs. 4 (b)

(i) Outline the steps involved in using the Durbin-Watson test for detecting model specification error(s).

(ii) Consider the following regression model for wages:

$$W_i = \alpha_1 + \alpha_2 Female_i + \alpha_3 Nonwhite_i + \alpha_4 Union_i + \alpha_5 Edu_i + \alpha_6 Exp_i + u_i$$

where W_i=wages, Edu_i= number of years of education, Exp_i= number of years of experience, Female_i=1 if female and 0 otherwise, Nonwhite_i=1 if nonwhite and 0 otherwise, and Union_i=1 if union member and 0 otherwise.

The regression model was estimated using data from 1289 observations. Using the estimated residuals from the model, the LM test for omitted variables was conducted to see if Exp_i^2 and the interaction between gender and experience $(Exper * Female)_i$ should be added to the

regression model. Describe the test procedure. If $R^2 = 0.0251$ for the auxiliary regression of the LM test, conduct the test at a 1% level of significance and state your conclusion.

(9.5+9.25)

Qs 5. (a) Consider the true model,

$$\widehat{CM}_i = 263.6416 - 0.0056PGNP_i - 2.2316FLR_i \ldots (1)$$

Where CM =Child mortality
PGNP = per capita GNP
FLR = Female Literacy Rate

Now, the estimated model is as below,

$$\widehat{CM}_i = 157.4244 - 0.0114PGNP_i \ldots . (2)$$

Where the slope coefficient in the regression of FLR on PGNP is 0.00256.

(i) Prove that the estimated parameters from the model (2) are biased as well as inconsistent.
(ii) Find the value of the specification bias due to an omission of the relevant variable, FLR?
(iii) Examine the variances of PGNP in the two models and discuss the trade-off between the bias and efficiency in this case.

Qs 5. (b) Consider the following function:

$$charity_{it} = B_1 + B_2 age_{it} + B_3 income_{it} + B_4 price_{it} + B_4 deps_{it} + B_4 MS_{it} + w_{it}$$

where charity - charitable contribution; price- the opportunity cost of giving charitable contributions; deps- no. of dependents; MS- marital status; and, $w_{it} = \varepsilon_i + u_{it}$

(i) Discuss the Random effects model in panel data analysis. State the assumptions made by this approach.
(ii) Is it true that the error term of a given cross-sectional unit at two different intervals are correlated?
(iii) What is the difference between a fixed and random effect model?
(iv) Following are the results of the Hausman test:

Correlated Random Effects - Hausman Test				
Test cross-section random effects				
Test summary	Chi-square statistic	Chi-sq d.f	Prob	
Cross-section random	15.964273	5	0.0069	
cross -section random effects test comparisons:				
Variable	Fixed	Random	Var (Diff.)	Prob.
age	0.102249	0.277063	0.003539	0.0033
income	0.838810	0.852996	0.000830	0.6224
price	0.366080	0.370199	0.000087	0.6595
deps	-0.086352	-0.036254	0.000487	0.0232
MS	0.199833	0.199669	0.016167	0.9990

State the null hypothesis of the Hausman test clearly. What can you conclude from the above result?

(9.5+9.25)

Qs. 6 (a) An OLS estimate of the rate of return to education is,

$$Lwage = \beta_0 + \beta_1 education + u$$

Suppose education and error term are related. A commonly used instrument for education is parental education i.e., mother's education.

(i) State and explain the conditions required for mother's education to be a good instrument for education. Using an example, explain the difference between a proxy variable and an instrumental variable.

(ii) Derive the IV estimator for β_1 and prove its consistency.

(iii) Suppose N=50, ΣZ= 50, ΣY= 150, ΣX= 150, ΣZX = 300, ΣZY= 200

Obtain the IV estimator of β_1.

Qs. 6 (b) Consider the following IS-LM model of macroeconomics:

LM equation

$$Y_t = \lambda_0 + \lambda_1 \bar{M} + \lambda_2 r_t + u_{1t}$$

IS equation

$$Y_t = \Pi_0 + \Pi_1 r_t + u_{2t}$$

Where Y_t = Income
r_t = Interest rate
$\bar{M}$ = assumed level of Money Supply

$E(u_{1t}) = 0$, $E(u_{2t}) = 0$, $E(u_{1t}^2) = \sigma_1^2$, $E(u_{2t}^2) = \sigma_2^2$, $E(u_{1t}u_{1t+j}) = 0$ (for $j \neq 0$), $E(u_{2t}u_{2t+j}) = 0$ (for $j \neq 0$), cov $(\bar{M}, u_{1t}) = 0$, $E(u_{1t}, u_{2t}) = 0$

(i) Are the OLS estimators for λ and Π coefficients consistent? Explain your answer.
(ii) Show that the covariances of the stochastic explanatory variable r_t with the disturbance terms, u_{1t} and u_{2t} are not equal to zero.

(9.5+9.25)