

Unique Paper Code : 62354343

Name of the Course : B.A. (Prog.) Mathematics

Name of the Paper : Analytic Geometry and Applied Algebra

Semester : III (CBCS)

Time : 3 Hours

Maximum Marks :75

- **Attempt any four questions in all.**
- **All questions carry equal marks.**

- 1) (a) Writing the basic steps, describe and draw the graph of the given equation showing their vertices, foci and asymptotes.

$$4(y-3)^2 - 9(x-2)^2 = 36$$

- (b) Identify and sketch the curve :

$$x^2 + 9y^2 + 2x - 18y + 1 = 0$$

- (c) Find an equation of the parabola that has its vertex at (1, 1) and directrix $y = -2$. Also, state the reflection property of parabola.

- 2) (a) Describe, sketch and label the focus, vertex and directrix of the parabola

$$2y^2 - 6y - 3x + 4 = 0$$

- (b) Find the equation of the ellipse whose axes are along the coordinate axes, vertices are $(\pm 5, 0)$ and foci at $(\pm 4, 0)$.

- (c) Find an equation for hyperbola that has same foci as the ellipse

$$12x^2 + 16y^2 = 48$$

and asymptotes $y = \pm \frac{2x}{3}$

- 3) (a) Find an equation of the largest sphere contained in the cube determined by the planes $x=2, x=16; y=4, y=18; z=7, z=21$.

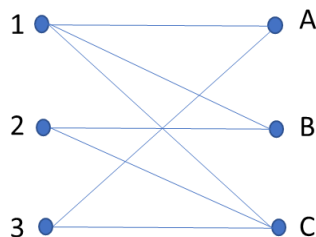
- (b) Rotate the coordinate axes to remove the xy -terms of the conic:

$$5x^2 - 6xy + 5y^2 - 49 = 0$$

Then name the conic.

- (c) (i) Find the vector of length 4 that makes an angle $\pi/6$ with positive x -axis.

- (ii) Find the angle between the vectors $\vec{u} = \hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{v} = 2\hat{i} + 7\hat{j} + 6\hat{k}$.
- 4) (a) Find the equation of sphere that has $(1, -2, 4)$ and $(3, 4, -12)$ as end points of diameter.
- (b) Given $\|\vec{a}\| = 10$, $\|\vec{b}\| = 2$ and $\vec{a} \cdot \vec{b} = 12$, find $\|\vec{a} \times \vec{b}\|$
- (c) (i) Using vectors find the area of triangle with vertices $A(1, 1, 1)$, $B(1, 2, 3)$ and $C(2, 3, 1)$.
- (ii) Use a scalar triple product to determine whether the vectors $\vec{u} = 4\hat{i} - 8\hat{j} - \hat{k}$, $\vec{v} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{w} = 3\hat{i} - 4\hat{j} + 12\hat{k}$ lie in the same plane.
- 5) (a) Show that the lines:
- $$L_1 : x = 1 + 4t, \quad y = 5 - 4t, \quad z = -1 + 5t$$
- $$L_2 : x = 2 + 8t, \quad y = 4 - 3t, \quad z = 5 + t$$
- Are skew lines and find the distance between them.
- (b) (i) Find the distance between the point $(1, -4, -3)$ and the plane $2x - 3y + 6z = -1$
- (ii) Determine whether the line:
 $L : x = 3 + 8t, y = 4 + 5t, z = -3 - t$ is parallel to the plane $x - 3y + 5z = 12$.
- (c) Find the volume of the tetrahedron with vertices $P(1, 2, 0)$, $Q(2, 1, 3)$, $R(-1, 0, 1)$ and $S(3, -2, 3)$
- 6) (a) A supermarket wishes to test the effect of putting cereal on five shelves at different heights. Show how to design such an experiment lasting five weeks and using five brands of cereal.
- (b) Find a matching for the following graph or explain why none exists



- (c) Find a minimal edge cover for the following graph. Given a detailed logical analysis.

